Paraconsistent and Paracomplete Zermelo-Fraenkel Set Theory

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We present a treatment of set theory in a four-valued *paracomplete* and *paraconsistent* logic, i.e., a logic where propositions can be neither true nor false, and can be both true and false. Our approach differs from most previous attempts at setting up a paraconsistent set theory, in that we are not interested in principles such as Full Comprehension or avoiding Russell's Paradox. Rather, we prioritise setting up a system with a clear ontology of non-classical sets that can be used to model incomplete and inconsistent phenomena.

We propose an axiomatic system BZFC, obtained by carefully analysing the intuition behind ZFC and translating the axioms appropriately. Our approach seems to overcome many of the obstacles encountered in previous attempts at such a formalization. Moreover, we introduce the *anti-classicality axiom* postulating the existence of non-classical sets, and prove a surprising results saying that the existence of a single non-classical set is sufficient to derive the existence of any other type of non-classical set.

We also look at equiconsistency results and translations between BZFC and classical ZFC set theory.